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A preliminary consideration on the origin of life as a cognitive system : Evolution from a simple harmonic oscillator system to chaotic learning oscillator machines

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Abstract: Origin of life was considered as a simplest neural network-like cognitive machine (1-4) capable of quasi-self-replicating, and preliminarily analyzed by mathematical modeling of harmonic oscillator quasi-replication systems. 2 x 2 matrix operators, F = $^t((a_{11}, a_{22}), (a_{21}, -a_{11}))$, where $a_{21} = -(a_{11}^2 + \omega^2)/a_{12}$. ($a_{ij}$ are real-number-valued) were used for realizing harmonic oscillation and for simulating simple learning neural network-machine capable of self-revising and generating earliest cognitive life. Dual oscillator system machine (represented by Lotka-Volterra system) possessing two angular frequencies with feedback functions was found to be generated by modifications of harmonic oscillator operators, and is capable of evolving to more complex cognitive life systems. All mathematical descriptions were made without using imaginary and/or complex numbers, which is achieved by using 2 X 2 matrix "real number-valued" operators. Hierarchical neural network modeling of harmonic oscillator operator was made and discussed.

Keywords: Quasi-self-replication, harmonic oscillation, neural-network of harmonic oscillation system

I. INTRODUCTION

A most important character of life is the ability to reproduce self-like systems in the next generation or in the next time-interval. Not only living individuals but also various bio-systems tend to reproduce self-like dynamic systems. Life has had evolved within its own environment, and therefore must have emerged by interacting with the environment, meaning that "complete" self-replication (= replication dependent exclusively on "self", and not on environment) would not have been a necessary condition both for the emergence of the earliest life and for later evolution of life. Such reproducing systems are here called "quasi-self-replication (QSR) system.

In order to theoretically analyze QSR system, we introduce a QSR operator, Q_s, satisfying

$$Q_s x = w x \qquad (0)$$

where $x = {}^t(x_1, x_2)$, and w is a constant.

However, quesi-self-replication for reproducing self-like systems have had been undoubtedly important for bio-evolution throughout origin and evolution of life.

II. MATHEMATICAL OPERATOR OF SELF-REPLICATION

Simple oscillation is a most simple harmonic oscillation described by a differential equation of x, as below;

$$D^2 x = -\omega^2 x, \qquad (1)$$

where D denotes differential operator, $D = d/dt$ (and therefore $D^2x = (d^2/dt^2)x$), and ω^2 (> 0) is a consonant or a function not dependent on t.

InEq.(1), ω is called angular frequency, and T given by

$$T = 2\pi/\omega \qquad (1a)$$

is called period. The solution of the differential equation, Eq.(1), is given by

$$x = Ae^{i\omega t} = A(\cos \omega t + i \sin \omega t),$$
$$= A \exp[-i \omega t] \qquad (2)$$

where i denotes imaginary unit. As is well-known, Eq.(2) represents uniform circular motion on complex plain with angular velocity, ω, where imaginary unit i is considered to be an operator for making a rotation of $\pi/2$ in the complex plain.

Harmonic oscillation system (1) creates a periodical spacio-temporal dynamics, and therefore is interesting from a viewpoint of the origin and evolution of quesi-self-replication biodynamic systems.

By considering the relations to Eq.(1), let F denote an operator, given by a 2 X 2 matrix,

$$F = (a_{ij}), \qquad (i,j = 1,2), \qquad (3)$$

which is also, in this paper, written by

$$F = {}^t((a_{11}, a_{12}), (a_{21}, a_{22})),$$
$$= {}^t((a_{11}, a_{12}),$$
$$(a_{21}, a_{22})), \qquad (3a)$$

where a_{ij} are constants or functions (either t-dependent or t-independent), satisfying

$$F^2 x = -\omega^2 x \qquad (4)$$

in which $x = {}^t(x_1, x_2) = {}^t(x_1(t), x_2(t))$, and ω is a real number-valued function or constant satisfying $\omega^2 > 0$ (for example, $\omega = \omega\,(x_1(t),\ x_2(t),\ t,\ dx_1/dt,\ dx_2/dt,\ d^2x_1/dt^2,\ d^2x_2/dt^2\ ),\ \omega=\omega_0\,(constant),$ *etc.*). Eq.(4) would therefore gives a generalization of harmonic oscillation system possessing *t*-dependent or *t*-independent ω.

Let us consider

$$Y = F\,x, \qquad (5)$$

$$Z = F\,y = F^2x, \qquad (6)$$

where $y = {}^t(y_1, y_2)$, and $z = {}^t(z_1, z_2)$, in which ^{t}(....) denotes transpose of vector (....). Then we have, from (3) and (4),

$$y_1 = a_{11}\,x_1 + a_{12}\,x_2 \qquad (7)$$

$$y_2 = a_{21}\,x_1 + a_{22}\,x_2 \qquad (8)$$

and

$$F^2 = B = (b_{ij}), \quad (i,j = 1,2) \qquad (9)$$

where

$$b_{11} = a_{11}^2 + a_{12}a_{21},$$
$$b_{12} = a_{12}(a_{11} + a_{22})$$
$$b_{21} = a_{21}(a_{11} + a_{22}),$$
$$b_{22} = a_{22}^2 + a_{12}a_{21}.$$

From Eq.(4) and (9), we have

$$Bx = -\,\omega^2 Ex, \qquad (9a)$$

where E denotes a 2 X 2 unit matrix.

Eq.(9a) is an eigenequation of B (= (b_{ij})), but in this section, we will analyze operator matrix $F = (a_{ij})$, satisfying

$$B = F^2 = -\,\omega^2 E, \qquad (9b)$$

which can be written as;

$$a_{11}^2 + a_{12}a_{21} = -\omega^2, \qquad (10)$$

$$a_{12}(a_{11} + a_{22}) = 0, \qquad (11)$$

$$a_{21}(a_{11} + a_{22}) = 0, \qquad (12)$$

$$a_{22}^2 + a_{12}a_{21} = -\,\omega^2, \qquad (13)$$

and satisfies Eq.(4).

By solving Eqs. (10)-(13), under the condition that x is a vector (= $^t(x_1,x_2)$) whose elements are real-number-valued, we find a sole solution for $a_{12}a_{21} \neq 0$, given by

$$F = F_1 = {}^t(\ (a_{11},\ a_{22}),\ (a_{21}\ ,\ -\ a_{11})),$$
$$\text{where } a_{21} = -\ (a_{11}^2 + \omega^2\ )/a_{12}\,. \qquad (14)$$

For the case of $a_{12}a_{21} = 0$, we have no solution for $F = (a_{ij})$ with real-number-valued elements.

A typical example of F_1, given by Eq. (14), where $a_{11} = 0$ and $a_{21} = \omega$ gives a generalized simple harmonic oscillation,

$$F_1 = {}^t(\ (\ 0\ ,\ -\omega\ ),\ (\omega\ ,\ 0\ )\ )$$
$$= \omega^2\ {}^t(\ (\ 0\ ,\ -\ 1\ ),\ (\ 1\ ,\ 0)\ ), \qquad (15)$$

from which we have

$$F_1^2\ x = -\ \omega^2\ x. \qquad (16)$$

In case of $F_1 = d/dt = D_1$, Eq.(16) means

$$D^2\ x = -\ \omega^2\ x, \qquad (16a)$$

which represents the well-known simple harmonic oscillation whose solution is given by

$$x_1 = r\cos(\omega t + b), \qquad (17a)$$

$$x_2 = r\sin(\omega t + b), \qquad (17b).$$

Eqs. (17a,b) correspond to the solution (2), without using imaginary unit, but written by two real variables x_1 and x_2.

Eq. (17) means that simple harmonic oscillation (even if ω is a constant) needs two real variables, and Eq.(17a) (without (17b)) cannot fully describe harmonic oscillation.

For the general case of $F = F_1$, given by Eq.(15), let us consider an operator given by

$$G = (g_{ij}) = e^{-tF} \qquad (18)$$
$$= \Sigma_{(k,\,0,+\infty)}\ c_k(tF)^k$$
$$= \Sigma_{(k,\,0,+\infty)}\ c_k t^k F^k$$
$$= \Sigma_{(m,0,+\infty)}\{c_{2m}t^{2m}F^{2m} + c_{2m+1}t^{2m+1}F^{2m+1}\}$$
$$= \Sigma_{(m,0,+\infty)}\{c_{2m}t^{2m}F^{2m} + c_{2m+1}t^{2m+1}F^{2m}\ F\}$$

where $c_k = 1/(k!)$, and $\Sigma_{(k,\ k',\ k'')}$ denotes summation from k =k' to k= k''.

Since $F^2 = {}^t((-\omega^2,\ 0),\ (0,\ -\omega^2)) = -\omega^2\ E$, where E denotes a 2 X 2 unit matrix operator., we have

$$F^{2m} = (\ -\omega^2\ E)^m = (-1)^m\omega^{2m}\,E\,, \qquad (19)$$

and then,

$$G = \Sigma_{(m,\,0,+\infty)}\{c_{2m}t^{2m}(-1)^m\omega^{2m}\,E$$
$$+\ c_{2m+1}t^{2m+1}\ (-1)^m\omega^{2m}\ EF\}$$
$$= \Sigma_{(m,0,+\infty)}\{(-1)^m c_{2m}(\omega t)^{2m}\,E$$
$$+\ (1/\omega)\ (-1)^m c_{2m+1}(\omega t)^{2m+1}\ F\}$$
$$= \{\Sigma_{(m,0,+\infty)}(-1)^m c_{2m}(\omega t)^{2m}\}E$$
$$+\ (1/\omega)\{\Sigma_{(m,0,+\infty)}(-1)^m c^{2m+1}(\omega t)^{2m+1}\}\ F$$
$$= (\cos\,\omega t)\ E + (1/\omega)\ (\sin\,\omega t)\ F.$$

Thus we find

$$G = (g_{ij}) = e^{-tF}$$
$$= (\cos\ \omega t\ )E + (1/\omega)(\sin\ \omega t)\ F \qquad (20)$$
$$= {}^t(\ (\cos\ \ \omega t + (a_{11}/\omega\ )\ \sin\ \omega t,\ \ (a_{12}/\omega)\ \sin\ \omega t\ ),$$
$$((a_{21}/\omega)\ \sin\ \omega t\ ,\ \cos\ \omega t\ -\ (a_{11}/\omega)\ \sin\ \omega t\ )), \qquad (21)$$
$$\text{where } a_{21} = -\ (\ a_{11}^2 + \omega^2\ )\ /\ a_{12}.$$

By using thus defined operator G, we easily find the two solutions (k = 1, 2) of Eqs. (5,6,14),

$$x(k) = {}^t\,(x_{k1},\ x_{k2}) = G\ {}^t\,(C_{k1},\ C_{k2})$$
$$= (g_{ij})\ (C_{k1},\ C_{k2})\ = e^{-tF}\ {}^t\,(C_{k1},\ C_{k2}) \qquad (22)$$
$$= {}^t(C_{k1}(\cos\ \omega t + (a_{11}/\omega)\ \sin\ \omega t + C_{k2}(a_{12}/\omega)\ \sin\ \omega t,$$
$$C_{k1}(a_{21}/\omega)\ \sin\ \omega t + C_{k2}\{\cos\ \omega t\ -\ (a_{11}/\omega)\sin\ \omega t\}), \qquad (23)$$

where C_{k1} and C_{k2} are constants (for $k = 1, 2$), and general solution is given by

$$x = A_1\,x(1) + A_2\,x(2)\,, \qquad (24)$$

where A_1 and A_2 are constants.

Thus we finally obtain Eq.(24), which are solutions of generalized harmonic oscillation given by Eqs.(5,6, and 9a), described by real number-valued variables.

A special case of $a_{11}=0$, *and* $a_{12}=\ \omega$, Eq.(23) is reduced to;

$$x(k) = {}^t(C_{k1}\ \cos\ \omega t + C_{k2}\ \sin\ \omega t,$$
$$-C_{k1}\ \sin\ \omega t + C_{k2}\ \cos\ \omega t\ ). \qquad (25)$$

On the other hand, from Eq.(9a) and Cramer's theorem, we have an eigenequation (of B);

$$\det\,(B + \omega^2 E) = 0, \qquad (26)$$

where $-\omega^2$ is eigenvalue of B. From Eq.(26), it follows that

$$\omega^4 + (a_{11}^2 + a_{22}^2 + 2\, a_{12}a_{21})\omega^2 + (a_{11}a_{22} - a_{12}a_{21})^2$$
$$= \{(\omega^2 - (a_{11}a_{22} - a_{12}a_{21})\}^2 + (a_{11} + a_{22})^2\omega^2,$$
$$= 0, \qquad (27)$$

and therefore we find

$$a_{11} + a_{22} = 0, \text{ and } \omega^2 = a_{11}a_{22} - a_{12}a_{21}. \qquad (28)$$

Eqs.(28) give; $\omega^2 = -(a_{11}^2 + a_{12}a_{21})$ (28a)
which correspond to Eq.(14).

Accordingly, harmonic oscillation Eq.(2) was described by using a vector comprising two variates x_1 and x_2, and 2 X 2 matrix operators, without using imaginary numbers and functions. It is important to note that two variates are needed for describing harmonic oscillation.

III. EVOLUTION from SIMPLE HARMONIC OSCILLATION to Lotka-Volterra-type OSCILLATION

$F = F_1$ given by Eq.(14) has a condition that $a_{22} = -a_{11}$, which is generally difficult to be exactly satisfied in biotic system, as found in Lotka-Volterra dynamics.

Here we consider

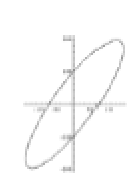
$$D\,x(t) = (d/dt)\,{}^t(dx_1/dt,\, dx_2/dt)$$
$$= {}^t(\,(a_{11}, a_{12}), (a_{21}, -a_{11} + \varepsilon)\,)\,x(t), \qquad (29)$$

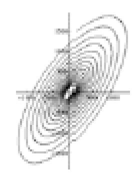
where $a_{21} = (a_{11}^2 + \omega^2)/a_{11}$. (29a)

If $\varepsilon = 0$ in Eq.(29), then we have $D^2 x = -\omega^2 x$, meaning that Eq.(29) is a harmonic oscillation.

(Ex.1) [Fig.1]: $\varepsilon = 0$, $a_{22} = -a_{11} = 1.2$, $a_{12} = -1$, $\omega^2 = 0.5$, $(x_1(0), x_2(0)) = (100, 50)$: As shown in Fig.1, the trajectory is an ellipsoid with axes. (abscissa: x_1), $x_1 = -(a_{12}/a_{11})\,x_2$, and $x_1 = -[a_{11}a_{12}/(a_{11}^2 + \omega^2)]\,x_2$.

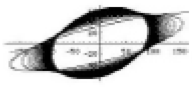
(Ex.2) [Fig.2], $\varepsilon = 0.05$, $a_{11} = 1.$, $a_{12} = -1$, $a_{21} = 2.$, $(x_1(0), x_2(0)) = (100, 50)$, $0 \le t \le 100$.: The trajectory





shows divergence.

Fig.1. Fig.2. (abscissa: x_1)



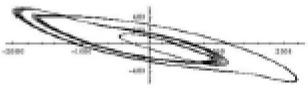


Fig. 3. Fig. 4

(Ex.3) [Fig.3] $\varepsilon = 0.$, $a_{11} = 1.2$, $a_{12} = -5.$, $\omega^2 = 0.5(1. + 0.3t)$, $(x_1(0), x_2(0)) = (200, 50)$, $0 \le t \le 80$.

(Ex.4) [Fig.4] $\varepsilon = -0.3a_{11}\mathrm{Sin}\ t$, $a_{11} = 1.2$, $a_{12} = -5.$, $\omega^2 = 0.5$, $(x_1(0), x_2(0)) = (200, 50)$, $0 \le t \le 150$. In this case, ε oscillates, and generates chaotic trajectory.

For considering generalization of harmonic oscillations, we need to consider another type of generalization of simple harmonic oscillation given by

$$H^2x = -W^2x = {}^t(-\omega_1^2x_1,\, -\omega_2^2x_2),$$
$$[\omega_1^2 > 0,\ \omega_2^2 > 0] \qquad (30.1)$$

where H and W are operators given by

$$H = {}^t(\,(h_{11}, h_{12}), (h_{21}, h_{22})), \qquad (30.2)$$
$$W = {}^t(\,(\omega_1, 0), (0, \omega_2), \qquad (30.3)$$

and $x = {}^t(x_1, x_2)$, in which all elements in H, W, and x are real-number-valued, and $\omega_1^2 > 0$, $\omega_2^2 > 0$. Eq.(4.1) needs to satisfy

$$h_{12}(h_{11} + h_{22}) = h_{21}(h_{11} + h_{22}) = 0, \qquad (31)$$

(Case 1) In case of $h_{12}h_{21} = 0$, we have H with real number-valued elements satisfying Eq.(30.1)

(Case 2) In case of $h_{12}h_{21} \neq 0$, *we have* $h_{11} + h_{22} = 0$ from Eq.(31), and from Eq.(30.1), finally find

$$\omega_1^2 = \omega_2^2, \qquad (31)$$

which is reduced to the case of Eq.(14).

Accordingly, we have reached a conclusion that we do not need to consider the cases given by Eq.(30.1).

IV. Lotka-Volterra-TYPE OSCILLATORS WITH TWO ANGULAR FREQUENCIES

The well-known Lotka-Volterra system is described by;

$$D\,{}^t(N_1, N_2) = (\,(r, -aN_1), (a'N_2, -r')\,{}^t(N_1, N_2), \qquad (32)$$

where $D = d/dt$. N_1 and N_2 denote number of prey and predator individuals, and a and a' are constants. Prey grow at a rate r in the absence of predator, and predators die at a rate of r'.

If $r = r'$, Eq. (32) is a case of Eq.(14), with an angular frequency ω, satisfying

$$a'N_2 = -(\omega^2 + r^2)/(-aN_1), \qquad (33)$$

and therefore ω^2 depends on N and N', as given by

$$\omega^2 = aa'NN' - r^2. \qquad (34)$$

In the more general cases of $r' \neq r$, Eq.(32) gives simultaneous non-linear differential equations.

For such cases we can make an oscillator model with two angular frequencies. By letting $F = (a_{ij})$ be an operator, we consider

$$F^2 x = (a_{ij})^2\,{}^t(x_1, x_2)$$
$$= ((-\omega_1^2, \varepsilon_{12}), (\varepsilon_{21}, -\omega_1^2\,))\,{}^t(x_1, x_2), \qquad (35)$$

And thereby we find

$$F^2\,{}^t(x_1, x_2) = ((-\omega_1^2,\ a_{12}(a_{11} + a_{22}),$$
$$(a_{21}(a_{11} + a_{22}),\ -\omega_1^2\,))\,{}^t(x_1, x_2). \qquad (36)$$

By letting $F = D = d/dt$, Eq.(35) is equivalent to;

$$D^2x_1 = -\omega_1^2\,x_1 + a_{12}(a_{11} + a_{22})\,x_2, \qquad (37a)$$
$$D^2x_2 = -\omega_2^2\,x_2 + a_{21}(a_{11} + a_{22})\,x_1, \qquad (37b)$$

which means harmonic oscillators under the existence of mutual external oscillating forces.

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V. COGNITIVE NEURAL NETWORK-LIKE STRUCTURE OF HARMONIC OSCILLATORS

Biosystems have cognitive structure, as has been widely discussed before (1-4). The most essential difference between life and non-life would be the existence of cognitive faculty in life, and therefore studies on the emergence of simplest cognitive system from non-cognitive system would be the most important viewpoint in studying the origins of life.

In order to solve such problems, we need to answer the question how first life could have had acquired cognitive structure and functions.

For example, a simple chemical reaction

$$===>X====>Y===>$$

with feedback regulation is very similar to simple harmonic oscillation, and the coupling of two such systems seems to generate a slightly more complex harmonic oscillation system like two- or three-layered neural network machine.

The harmonic oscillator of Eq.(14) is written in Fig.xx, where F is a harmonic operator given by $F = {}^t((a_{11}, a_{12}), (a_{21}, a_{22}))$, where $a_{22} = - a_{11}$, $a_{21} = - (a_{11}^2 + \omega^2)/a_{12}$. The left half in Fig.xx consists of three layers (x, Fx, F2x), where

$$F^k x_1 = a_{11} F^{k-1} x_1 + a_{12} F^{k-1} x_2$$
$$F^k x_2 = a_{21} F^{k-1} x_1 + a_{22} F^{k-1} x_2$$

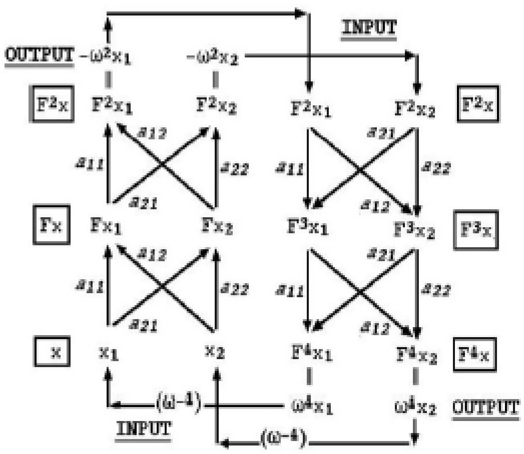

Fig. 4. Three-layerd Neural Network-like structure of generalized harmonic oscillator system, where $F = {}^t((a_{11}, a_{12}), (a_{21}, a_{22}))$, where $a_{22} = - a_{11}$, $a_{21} = - (a_{11}^2 + \omega^2)/a_{12}$. By letting $Q_s = F^4$, and $w = \omega^4$, we find that Q_s and w satisfy the Eq.(0), $Qs\, x = w\, x$, where Q denotes the above-defined quasi-replication operator. If $\omega^2 = 1$, then F^2 is a complete self-replication operator.

If we consider a_{ij} as connection coefficients (of neural network machine), then the left and right halves in Fig.xx are much like three-layered learning neural network machines with feedback functions. For the left side machine, the right side machine works as a feedback information-generating submachine, and for the right side machine, the left side machine works as a feedback submachine. The left and right side submachines constitute a whole possibly learning neural network-like quasi-replicative machine possibly capable of evolving to earliest cognitive biomachine (= life). Therefore, interactions between such hierarchical neural network quasi-replicative systems are good candidates for earliest cognitive systems being the first life.

Mathematical modeling of simplest cognitive system consisting of simple generalized harmonic oscillators would reveal essential characteristics of life as cognitive entity.

The most essential difference of organisms (biomachines) and man-made machines is the "teacher information" of (learning) machines. Organisms gain most important "teacher information" as their "internal self-information" by internalization of nearest neighborhood environments. For examples, lumens of guts, primary and secondary coeloms of multi-cellular animals, and ER (endoplasmic reticulum) of unicellular eukaryotes are, originally, both neighbouring environments of the ancestral organisms, and organisms needed to passively evolve under the control of "teacher information of the neighbouring environments". However, such environments' teacher information must have converted to "self-(teacher-)information after internalization of such environments into the body. Thus evolution frpm passive evolution to active evolution must have achieved by interacting behaviours of organism.

Faculties to achieve such "internalization of environment's teacher information to self-teacher information" would have been the key force for the emergence of autopoietic active evolution of life. Autopoiesis and actively evolving faculties of life are most plausibly based on the cognitive structure of life as analyzed in this and previous papers.

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